Solution Authoring Guidelines

Version 9.4
September 2016

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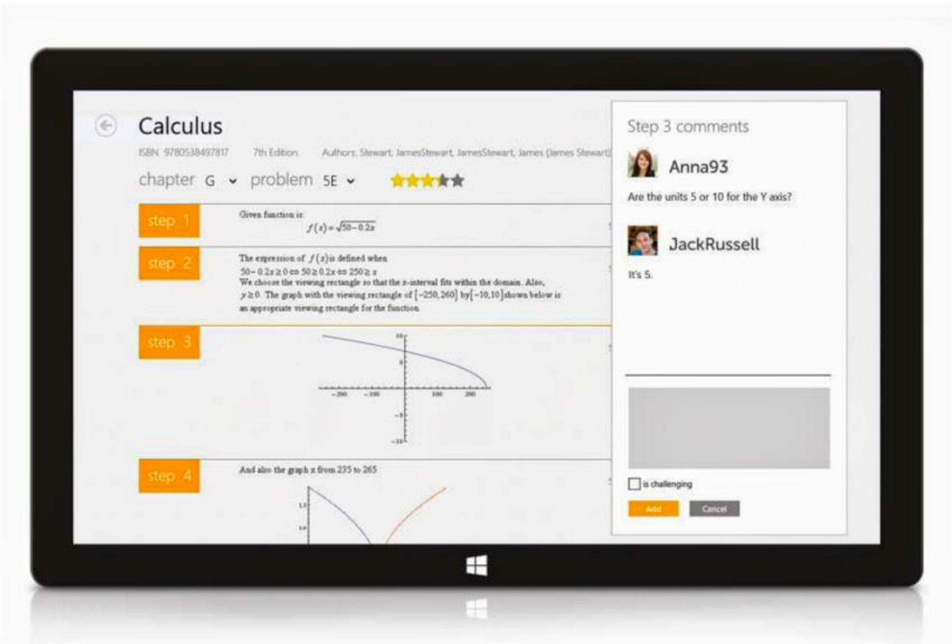

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Subject-specific Guidelines- Mathematics

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- Point related to Eigen values and Eigen vectors has been removed……………………………………………………………………………..Page no. 3
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List of changes made over Version 9.2

- Second point in M1.Content-A, related to blank type has been removed……………………………………………………………………………..Page no. 3

List of changes made over Version 9.3

1. Modified point 1 of A.Text/explanation under M1.Content…………………….3
2. Note added under C(iii) ………………………………………………………….5
3. First bullet point removed, second bullet point and sample example-Maple modified in C.Maple under M2 Technology…………………………………..…..10

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M1. Content:

A. Text/Explanation:

1. To prove a statement, bring out the generalization without sticking to any particular case of the statement.
2. Apply only the specified method to get the particular solution and do not use methods that are not explained in the textbook.

3. Iterative methods in numerical methods have to be clearly explained for one step from $y(x_0)$ to $y(x_n)$ using one step size (h) value. For other h values, a table must be given for other step size (h) values.

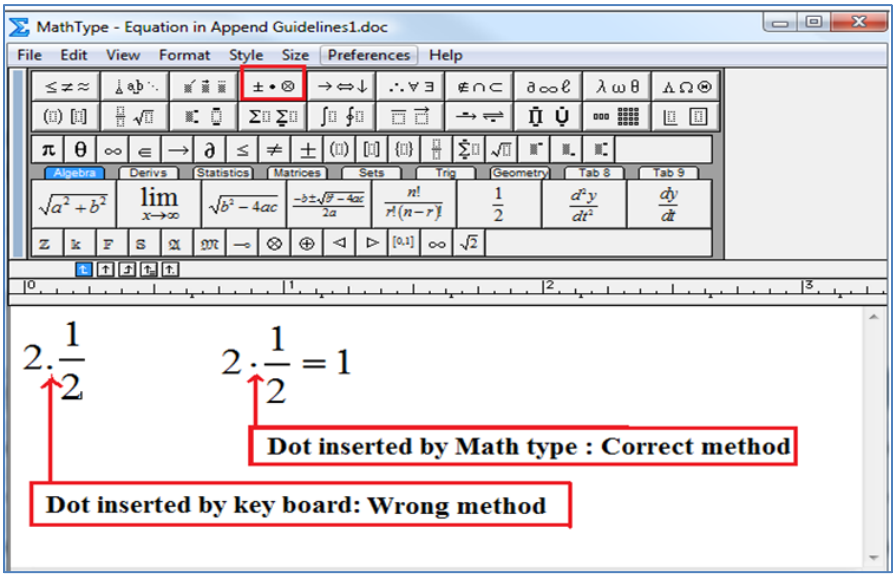
B. Diagrams/Graphs:

1. While drawing a curve, first explain the tracing process in the following manner: (i) scaling, (ii) angle in each quadrant in the case of polar curves along with the grid lines, (iii) the scale of the parameter in the case of parametric curve and, (iv) the domain of the function. The curve must be drawn only after providing these details.
2. Geometry-related problems must be supported by a graph or a figure if the problem demands.
3. Matching of figures and equations should be supported by reasons / definitions / statements in the textbook.
4. Matching of differential equation and its direction field requires the explanation of slopes of each solution curve at the initial points and critical points.
5. If question asked to match one equation with one of the given four options, then explain the correct option and also explain why the other options are incorrect.

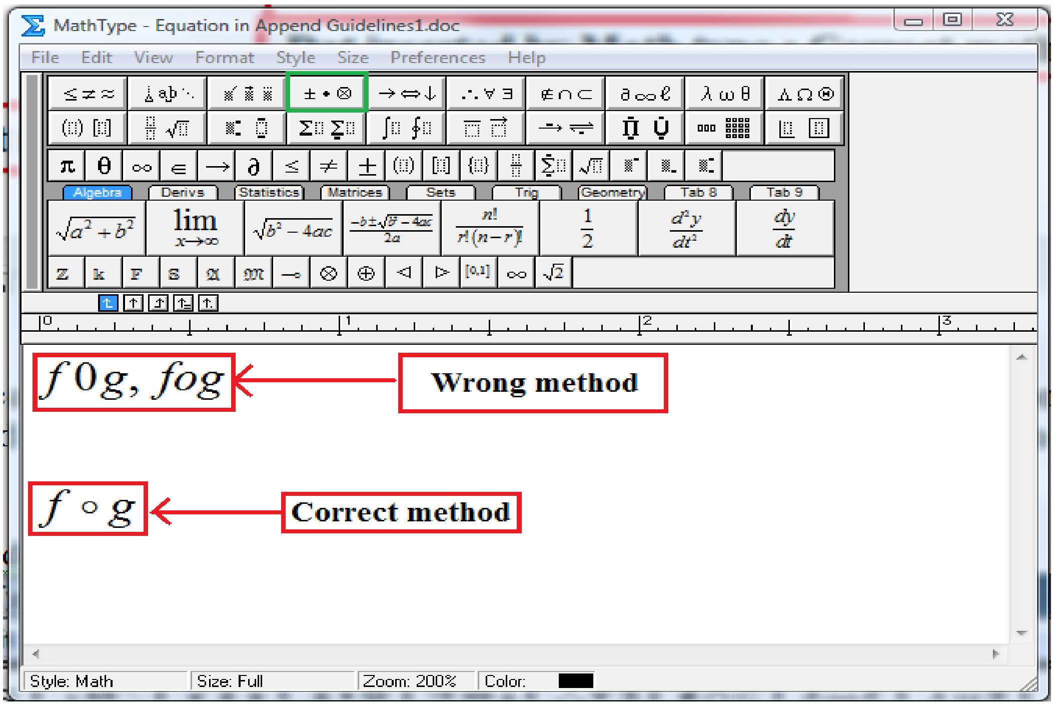
C. Equations:

<u>MathType/Microsoft Equation 3.0 Pointers</u>

(i) Do: $2\cdot\frac{1}{2}=1$ Don't: $2.\frac{1}{2}=1$



(ii) The composition of two functions f and g is denoted as $f \circ g$. The operational symbol '$\circ$' is available in the MathType / Microsoft Equation 3.0 menu.



(iii) Vector notation holds the cap symbol, bar or arrow upon the letter available in MathType.

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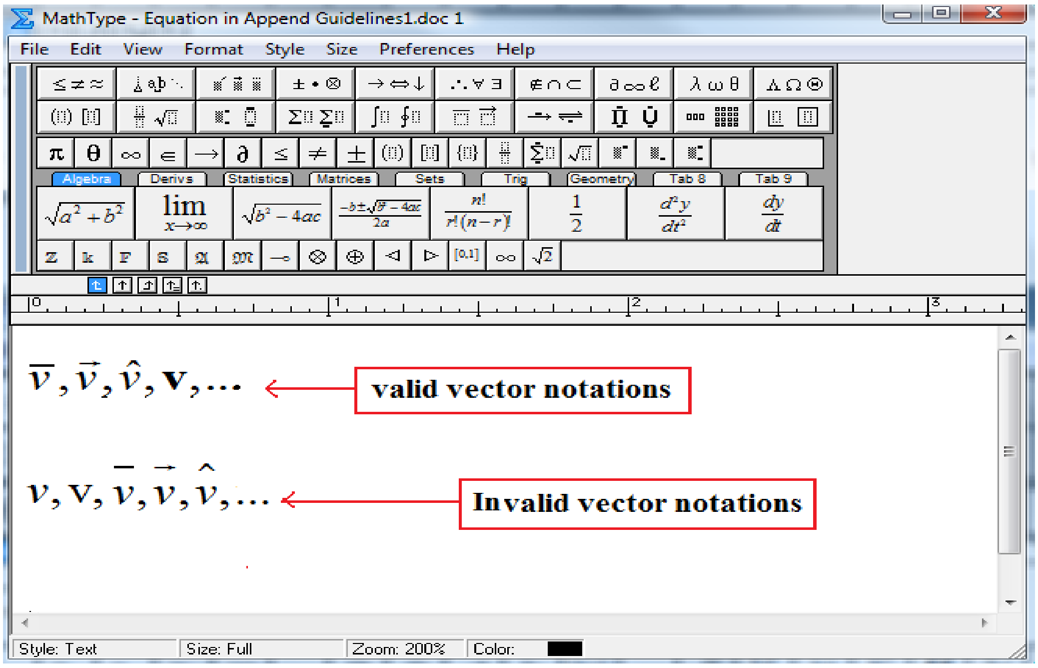
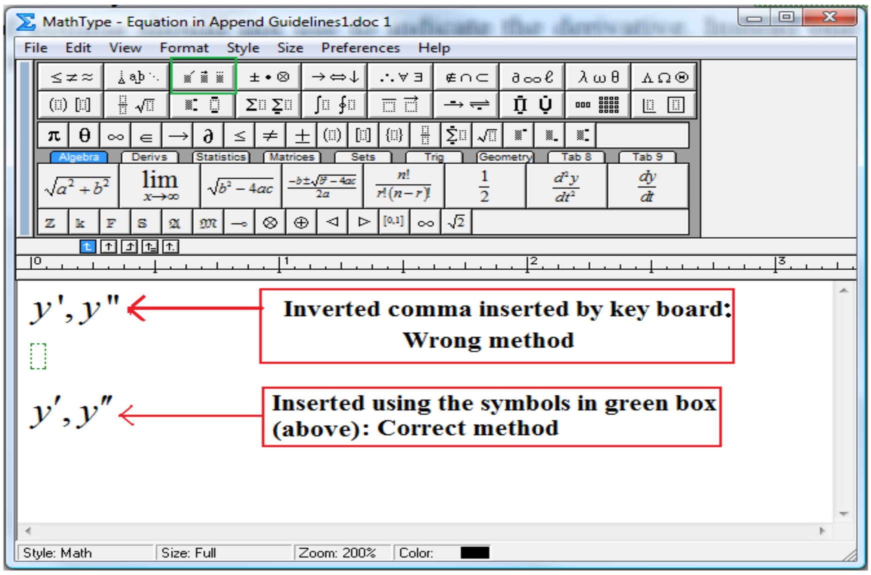


Note: Author should follow text book notations/symbols.

(iv) The available derivative symbol is shown in the green box below. For instance, (‘) inverted comma should not be used to indicate the derivative. Instead, one can use the derivative symbols shown here.



D. Tables:

1. Lengthy tables (more than 15 rows), should be broken into smaller tables (Approx. 10 rows each)

   Example:

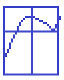
   A table containing 20 rows could be split into two tables with 10 rows each.

Right Approach:

S. No.	A	B	C	D
1	1	2	1	2
2	2	3	2	3
3	3	4	2	3
4	4	3	3	5
5	1	56	7	3
6	3	5	6	78
7	2	3	4	3
8	6	7	8	9
9	6	7	8	7
10	1	2	1	2

S. No.	A	B	C	D
11	2	3	2	3
12	3	4	2	3
13	4	3	3	5
14	1	56	7	3
15	3	5	6	78
16	2	3	4	3
17	6	7	8	9
18	6	7	8	7
19	6	7	8	7
20	6	7	8	7

M2. Technology:

Different types of icons such as , CAS, T, … indicate an exercise that definitely requires the use of either a Graphing Calculator or a Computer Algebra System(CAS) such as Matlab, Maple, Mathematica, etc.

In such cases, details of commands and outputs are required.

A. TI Calculators:

- Input keyboard strokes using MathType only.
- Solution must contain all the screenshots of the calculator showing outputs.

Question:

Use TI-84/89 calculator to verify $x = -14$ is the solution of $2(3x-2) = 4(3x+5) - 3(x-6)$.

Answer:

Take left side of the equation as $y_1 = 2(3x-2)$, and the right side of the equation as $y_2 = 4(3x+5) - 3(x-6)$.

In order to verify the solution, insert the functions y_1 and y_2 in the calculator and use the **Table** feature to verify the results.

First turn on the calculator and click on **Y =** to insert the functions against to Y_1 and Y_2.

Key strokes to insert the function $y_1 = 2(3x-2)$.

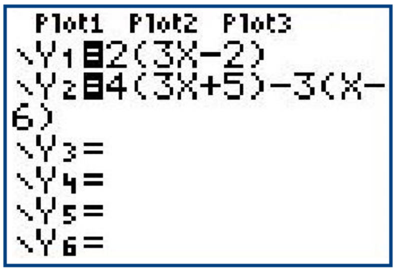
2 **(** **3** **X,T,θ,n** – 2 **)**

Click on the down arrow ▽ and insert the function y_2.

Key strokes to insert the function $y_2 = 4(3x+5) - 3(x-6)$.

4 **(** **3** **X,T,θ,n** + **5** **)** – **3** **(** **X,T,θ,n** **6** **)**

The display of the TI-calculator:



To determine the values of Y_1 and Y_2 for the corresponding X value, use the **Table** feature in the calculator.

Click on **2ND** and **WINDOW**, the TI calculator displays "**Table Setup**" window.

In this problem, X is independt variable and Y_1 and Y_2 are dependent variables on X.

Set **Ask** option for the independent variable, and **Auto** for the dependent varaible.

Use the arrow button ▷ to select the option **Ask** against the **Indpnt** and press **ENTER**, then select the option **Auto** against the **Depend** and press **ENTER**.

The display of the TI-calculator:

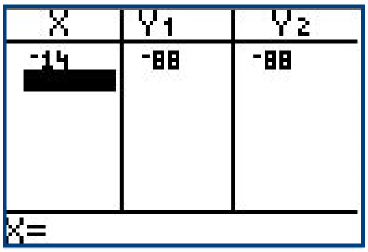
Click on $\boxed{\textbf{2ND}}$ + $\boxed{\textbf{GRAPH}}$ to get the **Table.**

The TI calculator displays a empty table which consists three columns for X, Y_1 and Y_2.
Enter the value for X as -14 and then press $\boxed{\textbf{ENTER}}$.

Key Strokes to insert the X value:

$\boxed{(-)}$ $\boxed{\textbf{1}}$ $\boxed{\textbf{4}}$

The TI calculator displays values of Y_1 and Y_2 as shown below.



Observe that, the values under Y_1 and Y_2 in the table for the corresponding $\mathrm{X}=-14$ are equal.
Thus, the function values of both sides of the equation are same.
Therefore, the solution set of the equation $2(3x-2)=4(3x+5)-3(x-6)$ is $\boxed{\{-14\}}$.

B. Mathematica:

- Input command should be in plain text, font 'Times New Roman', and font size 12.

- Copy the output from command window (Note book (.nb)) or take a screen shot of the output window and paste it in the solution.

SAMPLE SOLUTION – MATHEMATICA

Question:

Can there be beats when a damping force is added to the following model?

$$\frac{d^2x}{dt^2}+\omega^2 x=F_0\cos(\gamma t), x(0)=0, x'(0)=0.$$

Defend your position with graphs obtained either from the explicit solution of the problem

$$\frac{d^2x}{dt^2}+2\lambda\frac{dx}{dt}+\omega^2 x=F_0\cos(\gamma t),\ x(0)=0, x'(0)=0$$

or from solution curves obtained using a numerical solver.

Solution:

Investigate whether beats exist when a damping component is added to the model. Note that the following are constants: λ, ω, F_0, and γ.

$$\frac{d^2x}{dt^2}+2\lambda\frac{dx}{dt}+\omega^2 x=F_0\cos(\gamma t)$$

- The model has the initial conditions of $x(0)=0$ and $x'(0)=0$.
- Use the following values for the constants: $\lambda=0.01$, $\omega=2$, $F_0=1$, and $\gamma=22/9$. These values were set after doing trial and error for various values.
- Substitute the values in the differential equation, and use Mathematica to find a numerical solution.

Give the following commands as input using Mathematica software:

Input:

```
sol=NDSolve[{x''[t]+(2*0.01)*x'[t]+4*x[t]==Cos[(22/9)*t],x[0]==0,x'[0]==0},x,{t,0,40}]//Flatten
```

Output:

$\{x\to \text{InterpolatingFunction}[\{\{0.,40.\}\},"<>"]\}$

Input:

```
solution=x[t]/.sol
```

Output:

$\text{InterpolatingFunction}[\{\{0.,40.\}\},"<>"][t]$

Input:

```
Clear[x]
x[t_]:=Evaluate[solution]
x[t]
```

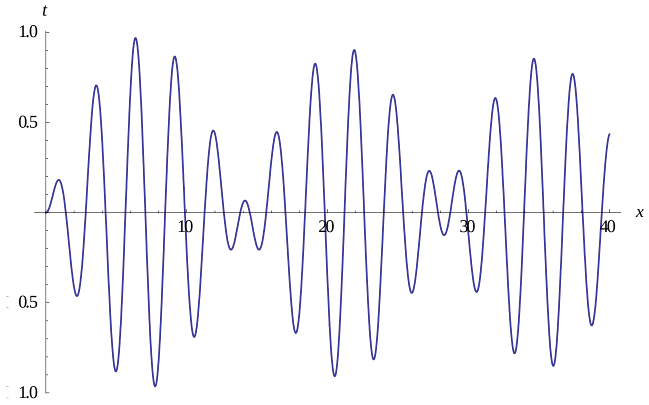
Output:

$\text{InterpolatingFunction}[\{\{0.,40.\}\},"<>"][t]$

Input:

grl=Plot[x[t],{t,0,40}, Axes->True, AxesLabel->{x,t}]

Output:



From the graph, it is apparent that beats still exist in the system despite the addition of the damping component.

C. Maple:

- To solve solutions using maple, enter the maple command and press “Enter” to get the output. Copy the command and the output from the command window (Maple document or work sheet) or take a screen shot of the output window and paste it in the solution.

SAMPLE SOLUTION – MAPLE

Question:

Use computer software to find a general solution of the differential equation,

$$Y''(t)-\frac{1}{t}\cdot Y'(t)+4t^3Y(t)=0, t>0$$

Solution:

Consider the differential equation,

$$Y''(t)-\frac{1}{t}\cdot Y'(t)+4t^3Y(t)=0, t>0$$

The objective is to find the general solution of the given differential equation by using Maple software.

In a new Worksheet, assign the given differential equation to the syntax ***ode*** by giving the below command.

Maple input:

$$> ode := \frac{d^2}{dt^2}y(t) - \left(\frac{1}{t}\right)\cdot\left(\frac{d}{dt}y(t)\right) + 4\cdot t^3\cdot y(t) = 0$$

Maple output:

$$ode := \frac{d^2}{dt^2}y(t) - \frac{\frac{d}{dt}y(t)}{t} + 4t^3 y(t) = 0$$

Use the ***dsolve*** command to solve the differential equation and with type mode '***series***' to get the series solution.

Maple input:

$$> ans := dsolve((ode), y(t), type='series')$$

Maple output:

$$ans := y(t) = \_C1\, t^2\left(1 - \frac{4}{35}t^5 + O(t^6)\right) + \_C2\left(-2 + \frac{8}{15}t^5 + O(t^6)\right)$$

Therefore, the required general solution is,

$$Y(t) = \boxed{C_1 t^2\left(1 - \frac{4}{35}t^5 + O(t^6)\right) + C_2\left(-2 + \frac{8}{15}t^5 + O(t^6)\right)}$$

D. Matlab:

- Input command should be in plain text, font 'Times New Roman', and font size 12.
- Copy the output from command window or take a screen shot of the output window and paste it in the solution.

Question:

Use MATLAB to request a row reduction of the matrix A, without showing intermediate steps and also find the rank of the following matrix

$$A = \begin{bmatrix} 2 & -3 & 0 & 1 & 4 \\ 1 & 4 & -6 & 3 & -2 \\ 0 & 11 & -12 & 5 & -8 \\ 4 & -1 & 5 & 3 & 7 \end{bmatrix}$$

Solution:

Consider the matrix A.

$$A = \begin{bmatrix} 2 & -3 & 0 & 1 & 4 \\ 1 & 4 & -6 & 3 & -2 \\ 0 & 11 & -12 & 5 & -8 \\ 4 & -1 & 5 & 3 & 7 \end{bmatrix}$$

Enter matrix A using MATLAB software.

Use the command **rref(A)** to obtain a matrix A_1, which is the row-reduced echelon form of matrix A.

Next, use the command **rank (A)** to obtain the rank of the matrix A.

Input:

```
A = [2 -3 0 1 4; 1 4 -6 3 -2; 0 11 -12 5 -8; 4 -1 5 3 7];
```

Output:

```
A =

     2    -3     0     1     4
     1     4    -6     3    -2
     0    11   -12     5    -8
     4    -1     5     3     7
```

Input:

```
rref(A)
```

Output:

```
ans =

    1.0000         0         0    0.9826    1.3217
         0    1.0000         0    0.3217   -0.4522
         0         0    1.0000   -0.1217    0.2522
         0         0         0         0         0
```

Thus, the row-reduced echelon form of A is,

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0.9826 & 1.3217 \\ 0 & 1 & 0 & 0.3217 & -0.4522 \\ 0 & 0 & 1 & -0.1217 & 0.2522 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The rank of the matrix A is found using the **rank(A)** command.

Input:

```
rank(A)
```

Output:

```
ans = 3
```

Therefore, the rank of the matrix A is $\boxed{3}$.

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Example solutions – Mathematics

List of changes made over Version 9.1

- Sample solution has been modified: Math Example 6(MCQ) ……….Page no. 169

List of changes made over Version 9.2

- Sample solution has been added: Math Example 6………..……….Page no. 27
- Sample solution has been changed: Math Example 7(MCQ)…………Page no. 27
- Sample solution has been added: Math Example 8(VSAQ)…..………Page no. 28
- Sample solution has been added: Math Example 9(FBQ)…..…………Page no. 29
- Sample solution has been added: Math Example 10(T/F)…..…………Page no. 30

List of changes made over Version 9.3

- All Sample solutions are modified.
- Sample solution has been added: Math Example 12 (Logic/Algorithmic type)… Page no.33

Math Example 1: Graphical type

Question: Sketch the graph of the curve $y = 8x^2 - x^4$.

Solution:

Consider the following function:

$y = 8x^2 - x^4$

Let $y = f(x)$, then $f(x) = 8x^2 - x^4$.

The objective is to sketch the graph of the given function by finding the intervals of increasing, decreasing, local maximum, minimum values, concavity and end behaviour.

(i)

To find the intervals of increasing and decreasing, differentiate the function with respect to x.

$f'(x) = 16x - 4x^3$ Since $\frac{d}{dx}(x^n) = nx^{n-1}$

Equate f' to 0, to get the critical points.

$$\begin{aligned} f'(0) &= 0 \\ 16x - 4x^3 &= 0 \\ 4x(4 - x^2) &= 0 \end{aligned}$$ Take out the common factor $4x$.

Notice that $f' = 0$, when $x = -2, x = 0$, or $x = 2$, so the critical numbers are $-2, 0,$ and 2.

Divide the domain into intervals whose endpoints are the critical numbers $-2, 0,$ and 2.

That is, $(-\infty, -2), (-2, 0), (0, 2)$ and $(2, \infty)$.

Take a test point in the intervals $(-\infty, -2), (-2, 0), (0, 2)$ and $(2, \infty)$ and construct the table to identify the intervals of increase or decrease.

For instance: Consider the interval**,** $(-\infty, -2)$:

Substitute -3 for x in the function $f'(x) = 16x - 4x^3$.

$$\begin{aligned} 16(-3) - 4(-3)^3 &= -48 + 108 \\ &= 60 \\ &> 0 \end{aligned}$$

As $f'(x)$ is positive $(+)$ in $(-\infty, -2)$, so the function f is increasing on $(-\infty, -2)$.

If $f'(x)$ is negative $(-)$ on an interval, then f is said to be decreasing on that interval.

Table shows the sing of $f'(x)$ on the specified intervals.

Interval	Test point	Sign of $f'(x)$	f
$(-\infty,-2)$	-3	+	Increasing on $(-\infty,-2)$
$(-2,0)$	-1	−	Decreasing on $(-2,0)$
$(0,2)$	1	+	Increasing on $(0,2)$
$(2,\infty)$	3	−	Decreasing on $(2,\infty)$

Therefore the function is increasing on $(-\infty,-2)$, $(0,2)$ and decreasing on $(-2,0)$, $(2,\infty)$.

(ii)

Find the local maximum and minimum values.
From the table, observe that $f'(x)$ changes from positive to negative at -2, so $f(-2)$ is the local maximum value. Similarly, $f'(x)$ changes from negative to positive at 0, so $f(0)$ is the local minimum value. Also, $f'(x)$ changes from positive to negative at 2, so $f(2)$ is the local maximum value.

Calculate the maximum and minimum values.

$$\begin{aligned} f(-2) &= 8(-2)^2 - (-2)^4 \\ &= 8(4) - (16) \\ &= 32 - 16 \\ &= 16 \end{aligned}$$

$$\begin{aligned} f(2) &= 8(2)^2 - (2)^4 \\ &= 8(4) - (16) \\ &= 32 - 16 \\ &= 16 \end{aligned}$$

$$\begin{aligned} f(0) &= 8(0)^2 - (-0)^4 \\ &= 8(0) - (0) \\ &= 0 - 0 \\ &= 0 \end{aligned}$$

Therefore local maximum value is $f(-2) = f(2) = \boxed{16}$ and local minimum value is $f(0) = \boxed{0}$

(iii)

Find the intervals of concavity and inflection points.
Calculate second derivative of the function $f(x) = 8x^2 - x^4$.

$f''(x) = 16 - 12x^2$

Here $f''(x)$ is defined everywhere, so equate $f''(x)$ to 0 to get the inflection points.

$$\begin{aligned}
16 - 12x^2 &= 0 \\
4(4 - 3x^2) &= 0 \\
(4 - 3x^2) &= 0 \\
3x^2 &= 4 \\
x^2 &= \frac{4}{3} \\
x &= \pm\frac{2}{\sqrt{3}}
\end{aligned}$$

Thus, $f''(x) = 0$ when $x = -\frac{2}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$.

Divide the domain into intervals with these numbers as endpoints.

$\left(-\infty, -\frac{2}{\sqrt{3}}\right), \left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$, and $\left(\frac{2}{\sqrt{3}}, \infty\right)$.

Determine the sign of f'' by substituting a value from each of the intervals to find the concavity of the graph.

For instance:

For $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$:

Substitute -2 for x in the function $f''(x) = 16 - 12x^2$.

$$\begin{aligned}
16 - 12(-2)^2 &= 16 - 12(4) \\
&= -32 \\
&< 0
\end{aligned}$$

As f'' is negative in $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$, so the function f is concave down on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$.

If f'' is positive on a particular interval then the function f is concave up on that interval.

Construct a table to discuss the concavity of function f in each interval.

Interval	Test point	Sign of $f''(x)$	Concavity
$\left(-\infty,-\frac{2}{\sqrt{3}}\right)$	-2	$-$	Downward
$\left(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$	0	$+$	Upward
$\left(\frac{2}{\sqrt{3}},\infty\right)$	3	$-$	Downward

Thus, f is concave upward on $\left(-\frac{2}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$ and concave downward on the intervals $\left(-\infty,-\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}},\infty\right)$.

The inflection points of the function are $x=-\frac{2}{\sqrt{3}}$ and $x=\frac{2}{\sqrt{3}}$, because f'' changes sign at these values. The corresponding inflection points are, $\boxed{\left(-\frac{2}{\sqrt{3}},\frac{80}{9}\right) \text{ and } \left(\frac{2}{\sqrt{3}},\frac{80}{9}\right)}$.

(iv)

Determine the end behaviour of the graph.

The end behaviour of the function means the behaviour of the function as $x\to\infty$ and $x\to-\infty$.
When $x\to\infty$, the value of the function is,

$$\begin{aligned}\lim_{x\to\infty}\left(8x^2-x^4\right)&=\lim_{x\to\infty}\left(\frac{8}{x^2}-1\right)x^4\\&=\boxed{-\infty}\end{aligned}$$

When $x\to-\infty$, the value of the function is

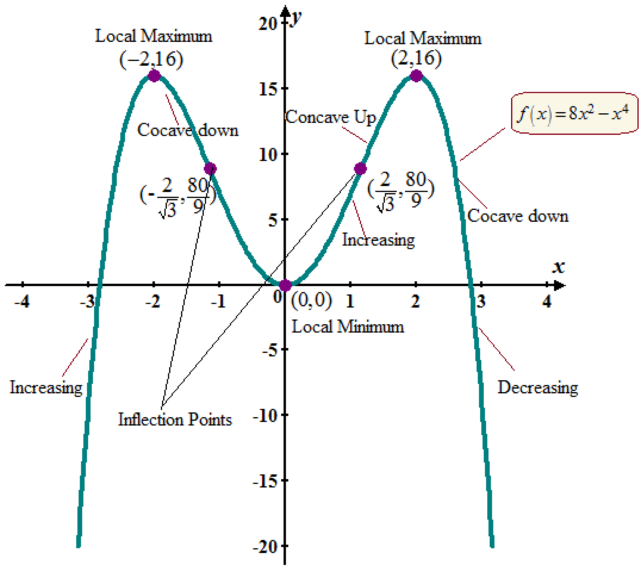
$$\begin{aligned}\lim_{x\to-\infty}\left(8x^2-x^4\right)&=\lim_{x\to-\infty}\left(\frac{8}{x^2}-1\right)x^4\\&=\boxed{-\infty}\end{aligned}$$

Therefore, the end behaviour of the function at $x\to\infty$ and $x\to-\infty$ is $\boxed{-\infty}$.

(v)

Using all above information, local maximum, minimum values, concavity and end behaviour, sketch the graph of the function.

The graph of the function $f(x) = 8x^2 - x^4$:



Math Example 2: Conceptual type

Question:

Use Euler's criterion to show that if p is an odd prime and a and b are positive integers not divisible by p, then $\left(\frac{a}{p}\right)\cdot\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$.

Solution:

The Euler's criterion states that if p is an odd prime and a is a positive integer not divisible by p, then the value of $\left(\frac{a}{p}\right)$ is as follows:

$$\left(\frac{a}{p}\right) \equiv a^{\frac{(p-1)}{2}} \pmod{p}$$

Here, p is an odd prime and that a and b are positive integers not divisible by p.

From Euler's criterion, write the relations for a and b.

$$\left(\frac{a}{p}\right) \equiv a^{\frac{(p-1)}{2}} \pmod{p} \text{ and } \left(\frac{b}{p}\right) \equiv b^{\frac{(p-1)}{2}} \pmod{p}$$

Result of Congruence Relation:
Let m be a positive integer. If $a \equiv b \pmod m$ and $c \equiv d \pmod m$, then, $ac \equiv bd \pmod m$.

Consider the following two relations for $\left(\frac{a}{p}\right)$:

$\left(\frac{a}{p}\right) \equiv a^{\frac{(p-1)}{2}} \pmod p$, and $\left(\frac{b}{p}\right) \equiv b^{\frac{(p-1)}{2}} \pmod p$

Use the congruence relation for the product of the two relations.

$$\begin{aligned}\left(\frac{a}{p}\right) \cdot \left(\frac{b}{p}\right) &\equiv a^{\frac{p-1}{2}} \cdot b^{\frac{p-1}{2}} \pmod p \\ &\equiv (ab)^{\frac{p-1}{2}} \pmod p \\ &\equiv \left(\frac{ab}{p}\right) \pmod p\end{aligned}$$

Thus,

$$\left(\frac{a}{p}\right) \cdot \left(\frac{b}{p}\right) \equiv \left(\frac{ab}{p}\right) \pmod p.$$

The objective is prove that,

$$\left(\frac{a}{p}\right) \cdot \left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right).$$

Use the contrapositive of required statement.

Suppose that,

$$\left(\frac{a}{p}\right) \cdot \left(\frac{b}{p}\right) \neq \left(\frac{ab}{p}\right)$$

The Legendre symbols $\left(\frac{a}{p}\right)$, $\left(\frac{b}{p}\right)$ and $\left(\frac{ab}{p}\right)$ take only the values 1 or –1.

If the left side value of the congruence takes the value 1, then the right side Legendre symbol should take -1 only.
That is, $1 \equiv -1 \pmod p$

Then, it becomes $2 \equiv 0 \pmod p$. This is not possible since p is greater than 2 and also an odd prime. Thus, the assumption was wrong.

Hence, it is proved that $\left(\frac{a}{p}\right) \cdot \left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$.

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Math Example 3: Table based

Question:

The symbol $\oplus$ denotes *exclusive or*, so $p \oplus q \equiv (p \vee q) \wedge \sim (p \wedge q)$. The truth table for *exclusive or* is as follows:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

a. Find simpler statement forms that are logically equivalent to $p \oplus p$ and $(p \oplus p) \oplus p$.

b. Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? Justify your answer.

c. Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? Justify your answer.

Solution:

Recall the definition of the expression $p \oplus q$ or p XOR q.
$p \oplus q = (p \vee q) \wedge \sim (p \wedge q)$, where $\oplus$ is denoted as "***exclusive or***".

Consider the following truth table for ***exclusive or*** :

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

(a)

Replace $\boldsymbol{q}$ by $\boldsymbol{p}$ by $\boldsymbol{p}$ in the definition,

$$\boldsymbol{p} \oplus \boldsymbol{p} = (\boldsymbol{p} \vee \boldsymbol{p}) \wedge \sim (\boldsymbol{p} \wedge \boldsymbol{p})$$

Use the results $\boldsymbol{p} \vee \boldsymbol{p} = \boldsymbol{p}$ and $\boldsymbol{p} \wedge \boldsymbol{p} = \boldsymbol{p}$. So, $\sim (\boldsymbol{p} \wedge \boldsymbol{p}) = \sim \boldsymbol{p}$

Substitute $\boldsymbol{p} \vee \boldsymbol{p} = \boldsymbol{p}$ and $\sim (\boldsymbol{p} \wedge \boldsymbol{p}) = \sim \boldsymbol{p}$ in $\boldsymbol{p} \oplus \boldsymbol{p} = (\boldsymbol{p} \vee \boldsymbol{p}) \wedge \sim (\boldsymbol{p} \wedge \boldsymbol{p})$.
$\boldsymbol{p} \oplus \boldsymbol{p} = \boldsymbol{p} \wedge \sim \boldsymbol{p}$

Use the ***negation law***: $\boldsymbol{p} \wedge \sim \boldsymbol{p} = \boldsymbol{c}$

Thus, $\boxed{p \oplus p = c}$.

Truth table for $p \oplus p$:

p	$p \oplus p$
T	F
F	F

Recall that disjunction and conjunction follow the associativity, and so the 'exclusive or' follows the associativity.

Use the result $p \oplus p = c$ to obtain,
$(p \oplus p) \oplus p = c \oplus p$.

According to the definition of the "***exclusive or***", $c \oplus p = (c \vee p) \wedge \sim (c \wedge p)$.

The ***identity law*** states that, $c \vee p = p$ and the ***universal bound law*** states that, $c \wedge p = c$.
Substitute $c \vee p = p$ and $c \wedge p = c$ in $c \oplus p = (c \vee p) \wedge \sim (c \wedge p)$.
$c \oplus p = p \wedge \sim c$
Use the result: Negation of c is t. That is, $\sim c = t$.
Substitute $\sim c = t$ in $c \oplus p = p \wedge \sim c$.
$c \oplus p = p \wedge t$
Use the ***identity law*** $p \wedge t = p$ to obtain,
$p \wedge t = p$

Therefore,
$\boxed{(p \oplus p) \oplus p = p}$

Truth table for $(p \oplus p) \oplus p$:

p	$p \oplus p$	$(p \oplus p) \oplus p$
T	F	T
F	F	F

(b)

Construct the truth tables for $(p \oplus q) \oplus r$, and $p \oplus (q \oplus r)$.

p	q	r	$p \oplus q$	$q \oplus r$	$(p \oplus q) \oplus r$	$p \oplus (q \oplus r)$
T	T	T	F	F	**T**	**T**
T	T	F	F	T	**F**	**F**
T	F	T	T	T	**F**	**F**
T	F	F	T	F	**T**	**T**
F	T	T	T	F	**F**	**F**
F	T	F	T	T	**T**	**T**
F	F	T	F	T	**T**	**T**
F	F	F	F	F	**F**	**F**

Observe that the two columns of $(p \oplus q) \oplus r$ and $p \oplus (q \oplus r)$ are equal in the table.

Hence, $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$.

(c)

Construct the truth tables for $(p \oplus q) \wedge r$ and $(p \wedge r) \oplus (q \wedge r)$.

p	q	r	$p \oplus q$	$(p \oplus q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \oplus (q \wedge r)$
T	T	T	F	**F**	T	T	**F**
T	T	F	F	**F**	F	F	**F**
T	F	T	T	**T**	T	F	**T**
T	F	F	T	**F**	F	F	**F**
F	T	T	T	**T**	F	T	**T**
F	T	F	T	**F**	F	F	**F**
F	F	T	F	**F**	F	F	**F**
F	F	F	F	**F**	F	F	**F**

Observe that, the two columns of $(p \oplus q) \wedge r$ and $(p \wedge r) \oplus (q \wedge r)$ are equal in the table.

Hence, $\boxed{(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)}$.

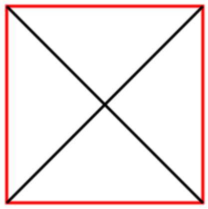
Math Example 4: Diagrammatic

Question:

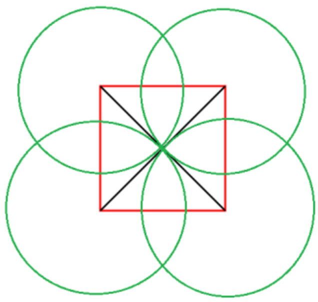
Sketch, label, and mark the figure for a pyramid with an octagonal base.

Solution:

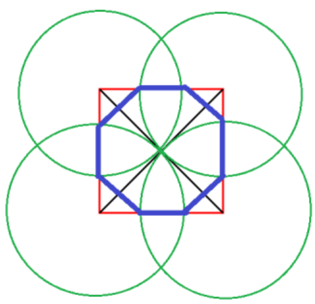
Follow the steps to sketch a pyramid with an octagonal base:
Step 1: First draw an octagon.
Step 2: Draw a square of any size on a sheet of paper; find the center of the square by drawing the line segments.



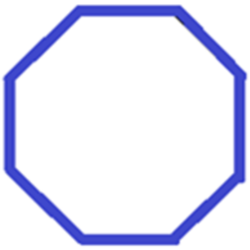
Step 3: Draw the four circles with their centers as vertices of the square; take the radius of the circles in such way that the circles must pass through the center of the square. Use compass to draw the circles.



Step 4: Join the all points where the circles are meeting the square with the line segments. The obtained figure is a regular octagon, highlighted in blue.

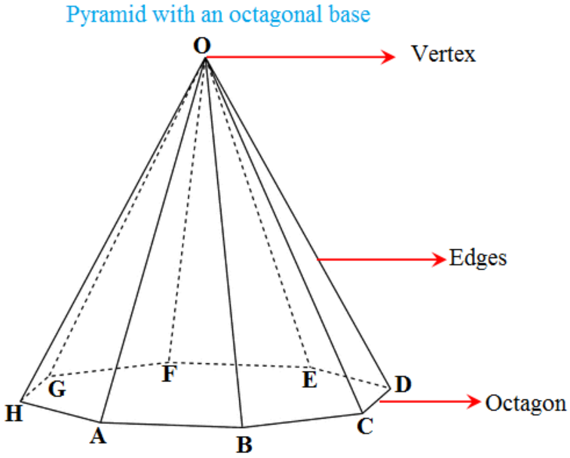


Step 5: Now the closed figure of a regular octagon is drawn separately.



Step 6: To make a pyramid with an octagonal base on sheet of paper, draw a regular octagon, mark the vertices as A, B, C, D, E, F, G and H. Mark the point O on top of the octagon.

Step 7: Now join the point O with the points A, B, C, D, E, F, G, and H with help of line segments.

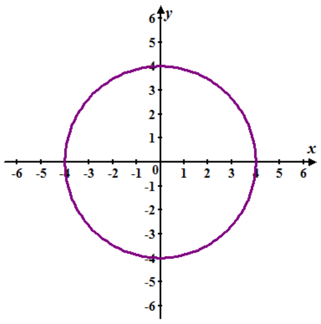


Math Example 5: Multiple choice

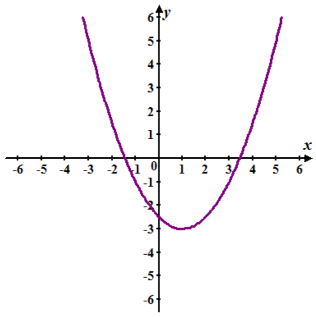
Question:

Match the equation $(y+2)^2 = 4(x-2)$ with one of the graphs (a) to (d), which follows:

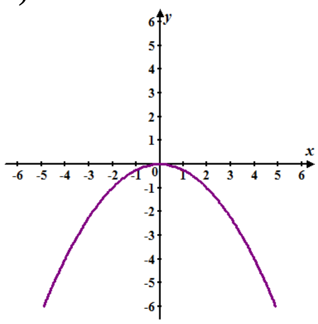
a)



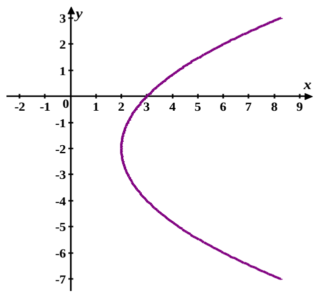
b)



c)



d)



Solution:

Consider the equation:

$$(y+2)^2 = 4(x-2)$$

As this equation represents a parabola, eliminate the option (a) which represents a circle.

Compare the equation with the standard equation of the parabola $(y-k)^2 = 4p(x-h)$.

Here, the vertex is (h,k), and the focus is $(h+p,k)$.

Rewrite the equation in standard form.

$$(y+2)^2 = 4\cdot 1\cdot(x-2)$$

Thus, $h=-2, k=2,$ and $p=1$.

Therefore, the vertex of the parabola is $(2,-2)$, and the focus is $(2+1,-2)=(3,-2)$.

Use this vertex to choose the correct option and eliminate the remaining.

In option (b), the vertex of the parabola is located at $(1,-3)$, so that this can be eliminated.

Similarly in option (c), the vertex is $(0,0)$, so that this can be eliminated.

Finally in the option (d), the vertex is located at $(2,-2)$.

Hence, the graph of the equation $(y+2)^2 = 4(x-2)$ matches with option $\boxed{\textbf{(d)}}$.

Math Example 6: Very Short answer type

Question

Express the verbal model in symbols, "*A* varies directly as the square of *p*".

Solution

The objective is to rewrite the verbal model in terms of symbols.
Here, the variables are A and *p*.

Use the following definition of direct variation:
The phrase ' ' y varies directly as x ' ' means that $y = kx$ for some nonzero constant k.

Square of p is p^2 and A varies directly as the square of p .
Using above definition write the verbal model in symbols
$A \alpha p^2$ Or $A = kp^2$
Here k is the constant of proportionality $\boxed{A = kp^2}$

Math Example 7: Fill in the blank type

Question

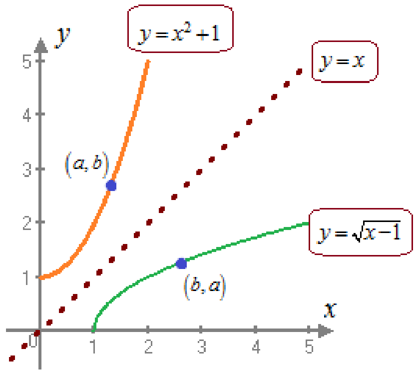
The graphs of f and f^{-1} are _____ with respect to the line $y = x$.

Solution

Reflexive property of inverse functions says that, the one-to-one function f and its inverse f^{-1} are symmetric about the line $y = x$, and they are mirror images.

According to this, if (a,b) is a point on the graph of f, then the point (b,a) should lie on its inverse function f^{-1}, and vice-versa.

For instance:



Therefore, the blank can be filled with $\boxed{\text{reflections}}$.

Math Example 8: True or False type

Question

Both $-3-i\sqrt{5}$ and $3-i\sqrt{5}$ are solutions to $P(x)=5x^3-9x^2+17x-23=0$.
Is it True or False?

Solution

Let $P(x)=5x^3-9x^2+17x-23=0$ be a polynomial equation.

Observe that this equation has degree 3 and it has real coefficients.

Use **Conjugate Pairs Theorem** statement, "if $P(x)=0$ is a polynomial equation with real coefficients and the complex number $a+ib$ is a root, and then $a-ib$ is also a root."

That is, if the polynomial equation has real coefficients, then imaginary roots occur in conjugate pairs.

If both $-3-i\sqrt{5}$ and $3-i\sqrt{5}$ are the solutions to the polynomial equation $P(x)=0$, then by **Conjugate Pairs Theorem** their corresponding conjugates $-3+i\sqrt{5}$ and $3+i\sqrt{5}$ are also solutions to the polynomial equation. So, there are 4 roots to $P(x)=0$.

This is impossible because the degree of the polynomial $P(x)$ is 3.

Hence, the given statement is $\boxed{\textbf{false}}$.

Chegg

Math Example 9: Check part type

Question

Solve the equation $4c-7=-c+3$ and **check** the solution.

Solution:

Consider the equation:
$4c-7=-c+3$.

The objective is to solve the equation for c and to check back the solution for correctness.

Use the properties of equality and inverse operations to solve the equation.

$4c-7+c=-c+3+c$	Add c to both sides of the equation
$5c-7=3$	Combine the like terms
$5c-7+7=3+7$	Add 7 to both sides of the equation
$5c=10$	Combine the like terms
$\frac{5c}{5}=\frac{10}{5}$	Divide both sides of the equation by 5
$c=2$	Perform the division

The solution of $4c-7=-c+3$ is $c=2$.

Check:

Check the solution by substituting it into the original equation.

Substitute $c=2$ in the equation $4c-7=-c+3$.

$4(2)-7\stackrel{?}{=}-2+3$	Replace c with 2
$8-7\stackrel{?}{=}1$	Simplify
$1=1$	**True**

Thus, the solution of $4c-7=-c+3$ is $\boxed{c=2}$.

Math Example 10: Dosage calculation type

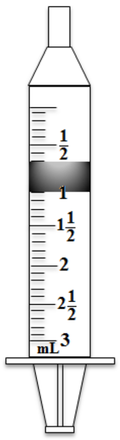
Question:

Determine the dosage measured by the sketched syringe.

Solution:

Objective of the problem is to determine the dosage measure by the sketched syringe.

The measured 3 mL syringe is sketched as shown in the below figure:



Observe that the calibrations used in the figure are for the metric mL scale, and the longer calibrations differ from $\frac{1}{2}$ mL to each other.

It means that each calibration measures 0.1 mL.

From the figure, the rubber tip is at 7 calibrations from the syringe hub. Since, each calibration measures 0.1 mL, the dosage must be measured 0.7 mL.

Therefore, the dosage measured by the sketched syringe is $\boxed{0.7 \text{ mL}}$.

Math Example 11: Application type

Question:

A rectangular piece of cardboard is twice as long as it is wide. A 4-cm square is cut out of each corner, and the sides are turned up to make a box with an open top. The volume of the box is 616 cm^3. Find the original dimensions of the cardboard.

Solution:

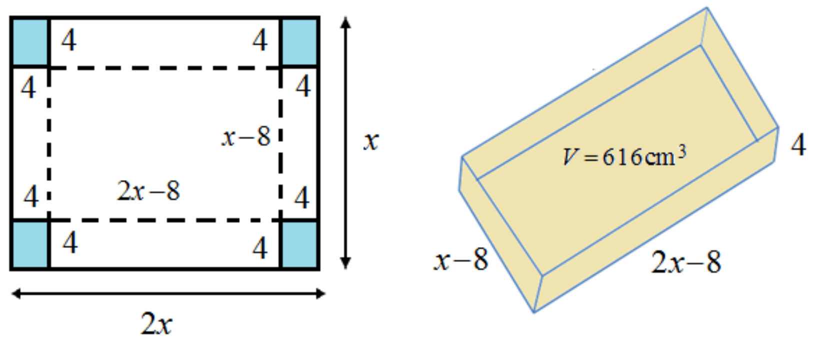
The length of the rectangular piece of cardboard is twice its width.

The volume of the box is $616\,cm^3$.

Let x be the width of the rectangular piece of card board (*l*), in centimetres.

Then the length of the rectangular piece of card board is $2x$.

A 4cm square is cut out of each corner of the cardboard sheet and then the remaining flaps are folded up to form a box as shown in the following figure:



Therefore, the length of the box is $2x-8$ and the width is $x-8$.

Volume of the Rectangular box = Length $\times$ Width $\times$ Height

$$=(2x-8)(x-8)4$$

Given that volume of the box is $616\ cm^3$.

Thus,

$(2x-8)(x-8)4=616$

Solve the equation for x

$$
\begin{aligned}
4(2x-8)(x-8)&=616 && \\
(2x-8)(x-8)&=154 && \text{Divide both sides by 4}\\
2x^2-16x-8x+64&=154 && \text{Multiply the polynomials}\\
2x^2-24x+64&=154 && \text{Combine like terms on left side}\\
2x^2-24x-90&=0 && \text{Subtract 154 from both sides}\\
(x-15)(2x+6)&=0 && \text{Factor}\\
x-15=0 \text{ or } 2x+6&=0 && \text{Zero-factor property}\\
x=15 \text{ or } x&=-3 &&
\end{aligned}
$$

The solutions of the equation are 15 and -3.
Since the width must be positive, -3 cannot be a solution.
So, the width of the cardboard, x = 15cm.
Therefore, the length of the cardboard is,

$$\begin{aligned} 2x &= 2(15\text{cm}) \\ &= 30 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{The volume of the box} &= 4(2x-8)(x-8) \\ &= 4(2\cdot 15-8)(15-8) \\ &= 4(30-8)(15-8) \\ &= 4(22)(7) \\ &= 616 \end{aligned}$$

This agrees with the information given in the problem.
Therefore, the length of the cardboard is $\boxed{30 \text{ cm}}$ and width of the cardboard is $\boxed{15 \text{ cm}}$.

Math Example 12: Logic/Algorithm type

There is no way to change the quantifiers or symbolic nations in the logic/algorithm based solutions, so it is suggested to add more explanation before write the symbolic statements or algorithm. This helps to get rid of plagiarism.

Question:

There are at least two attorneys in the office. All attorneys are professionals.

There are at most two professionals in the office. Therefore, there are exactly two professionals in the office. ($Ax: x$, is an attorney; $Ox: x$, is in the office; $Px: x$, is a professional)

Solution:

Consider the first statement:

"There are **at least** two attorneys in the office."

Use the existential quantifier $(\exists)$, to translate "**at least**" statement.

Let x and y be two attorneys in the office.

There are at least two attorneys, thus means those two are different. That is, $x \neq y$

Suppose that, Ax, Ay represent that x, y are attorneys and Ox, Oy are represent that the x, y are in the office.

Thus, the given statement can be written by quantifiers as below:

1. $(\exists x)(\exists y)(Ax \bullet Ox \bullet Ay \bullet Oy \bullet x \neq y)$

Consider the second statement:

"**All** attorneys are professionals."

Use the quantifier $\left((\quad)\right)$, to translate "**All**" statement.

Suppose that, Px is represents that the attorney x is professional.

The given statement gives that; the set of all attorneys are professionals.

Thus, the given statement can be written by quantifiers as below:

2. $(x)(Ax \supset Px)$

Consider the third statement:

"There are **at most two** professionals in the office"

The conclusion contains "at most two", so that needs to take three quantifiers; and so on.

3. $(x)(y)(z)\left[(Px \bullet Ox \bullet Py \bullet Oy \bullet Pz \bullet Oz) \supset (x = y \vee z = z \vee x = z)\right]$

The conclusion part, states that “There are **exactly two** professionals in the office.”

Here, x and y are two professional in the office $(x \neq y)$.

As per conclusion, if there are another professional exists in the office then he/she would be either x or y.

Therefore, the conclusion can be written as:
$(\exists x)(\exists y)\{Px \bullet Ox \bullet Py \bullet Oy \bullet x \neq y \bullet (z)[(Pz \bullet Oz) \supset (z = x \vee z = y)]\}$

The objective is to derive the conclusion of the given arguments by using conditional proof or indirect proof.

Remove one existential quantifier by applying the rule “Existential instantiation” on 1 and remove $\exists x$.
4. $(\exists y)(Aa \bullet Oa \bullet Ay \bullet Oy \bullet a \neq y)$

Remove one existential quantifier by applying the rule “Existential instantiation” on 4 and remove $\exists y$.
5. $Aa \bullet Oa \bullet Ab \bullet Ob \bullet a \neq b$

Remove the universal quantifier by applying a valid rule of inference on 2, which is “Universal instantiation”.
6. $Aa \supset Pa$
7. Aa Simplification on 5

Use Modus Ponens rule $p \supset q / p // q$ for steps 6 and 7.
8. Pa
Remove the universal quantifier by applying a valid rule of inference on 2, which is “Universal instantiation”.
9. $Ab \supset Pb$
Rearrange the conjunctions or disjunctions by using commutative on 5.
10. $Ab \bullet Ob \bullet a \neq b \bullet Aa \bullet Oa$
11. Ab Simplification by 10
Use Modus Ponens rule $p \supset q / p // q$ for steps 9 and 11.
12. Pb
Rearrange the conjunctions by using commutative on 10.
13. $Oa \bullet Ab \bullet Ob \bullet a \neq b \bullet Aa$
14. Oa Simplification by 13
Again, rearrange the conjunctions by using commutative on 5.
15. $Ob \bullet a \neq b \bullet Aa \bullet Oa \bullet Ab$

16. Ob Simplification by 15

Again, rearrange the conjunctions by using commutative on 5.

17. $a \neq b \bullet Aa \bullet Oa \bullet Ab \bullet Ob$

18. $a \neq b$ Simplification by 17

Use dot operator and conjunct the statements 8, 12, 14, 16, and 18.

19. $Pa \bullet Oa \bullet Pb \bullet Ob \bullet a \neq b$

The symbolic form of conclusion is:

$(\exists x)(\exists y)\{Px \bullet Ox \bullet Py \bullet Oy \bullet x \neq y \bullet (z)[(Pz \bullet Oz) \supset (z = x \vee z = y)]\}$

To start with Indirect Proof, first start by assuming the negation (AIP) of the statement to be conclude/obtained.

20. $\sim (z)[(Pz \bullet Oz) \supset (z = a \vee z = b)]$

Introduce the quantifier $\exists$, by applying Quantifier negation rule on 20.

21. $(\exists z) \sim [(Pz \bullet Oz) \supset (z = a \vee z = b)]$

Remove one existential quantifier by applying the rule "Existential instantiation" on 21 and remove $\exists z$ and then apply implication on obtained one.

22. $\sim [(Pc \bullet Oc) \supset (c = a \vee c = b)]$

Use Material implication formula $(p \supset q) :: (\sim p \vee q)$ on 22.

23. $\sim [\sim (Pc \bullet Oc) \vee (c = a \vee c = b)]$

To allow tildes to be moved inside and outside of parentheses, use the De Morgan's law on 23.

24. $\sim\sim (Pc \bullet Oc) \bullet \sim (c = a \vee c = b)$

Delete the pair of negations by applying Double Negation rule $\sim\sim p :: p$ on 24.

25. $Pc \bullet Oc \bullet \sim (c = a \vee c = b)$

Remove the Universal quantifiers from 3 by applying Universal Instantiation repeatedly.

26. $(y)(z)[(Pa \bullet Oa \bullet Py \bullet Oy \bullet Pz \bullet Oz) \supset (a = y \vee a = z \vee y = z)]$

27. $(z)[(Pa \bullet Oa \bullet Pb \bullet Ob \bullet Pz \bullet Oz) \supset (a = b \vee a = z \vee b = z)]$

Universal Instantiation from 26

28. $(Pa \bullet Oa \bullet Pb \bullet Ob \bullet Pc \bullet Oc) \supset (a = b \vee a = c \vee b = c)$

Universal Instantiation from 27

Apply the simplification rule on 25.

29. $Pc \bullet Oc$

30. $Pa \bullet Oa \bullet Pb \bullet Ob$ Simplication on 19

Use dot operator and conjunct the statements 29 and 30.

31. $Pa \bullet Oa \bullet Pb \bullet Ob \bullet Pc \bullet Oc$ Conjunction of 29 and 30

Use Modus Ponens rule $p \supset q / p // q$ for steps 28 and 31.

32. $a=b \vee a=c \vee b=c$ Modus Ponens for 28 and 31

From 18, $a \neq b$ and 32 states that $a=b \vee a=c \vee b=c$, so there is a disjunctive Syllogism $(p \vee q / \sim p // q)$ for both 18 and 32.

33. $a=c \vee b=c$

Rearrange the conjunctions by using commutative on 25.

34. $\sim(c=a \vee c=b) \bullet Pc \bullet Oc$

35. $\sim(c=a \vee c=b)$ Simplification on 34

Translate the statement 35 by using identity $x=y :: y=x$.

36. $\sim(a=c \vee c=b)$

Translate the statement 35 by using identity $x=y :: y=x$.

37. $\sim(a=c \vee b=c)$

Use dot operator and conjunct the statements 33 and 37.

38. $(a=c \vee b=c) \bullet \sim(a=c \vee b=c)$

As this is not true, it contradicts the assumption of the indirect proof for statement 20.
So by the indirect proof, write the statement as:

39. $\sim\sim(z)[(Pz \bullet Oz) \supset z=a \vee z=b]$

Delete the pair of negations by applying Double Negation rule on 39.

40. $(z)[(Pz \bullet Oz) \supset z=a \vee z=b]$

Use dot operator and conjunct the statements 19 and 40.

41. $Pa \bullet Oa \bullet Pb \bullet Ob \bullet a \neq b \bullet (z)[(Pz \bullet Oz) \supset (z=a \vee z=b)]$

Introduce the existential quantifiers by apply the "Existential generalization" rule on 41, and apply on the obtained statement also.

42. $(\exists y)\{Pa \bullet Oa \bullet Py \bullet Oy \bullet a \neq y \bullet (z)[(Pz \bullet Oz) \supset (z=a \vee z=y)]\}$

43. $(\exists x)(\exists y)\{Px \bullet Ox \bullet Py \bullet Oy \bullet x \neq y \bullet (z)[(Pz \bullet Oz) \supset (z=x \vee z=y)]\}$

Existential Generalization from 42

Here, the obtained statement 43 is same as the conclusion.

Hence, the conclusion is derived from the arguments by using conditional proof or indirect proof.

Back